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LETTRES EPISTEMOLOGIQUES

EPISTEMOLOGISCHE BRIEFE

Hidden Variables and Quantum Uncertainty
(Written Symposium, 35th Issue)

Variables cachées et indéterminisme quantique
(Symposium écrit, 35ème livraison)

Verborgene Parameter und Quanten-Unbestimmtheit
(Schriftliches Symposium, 35.Heft)

December 1983 Décembre

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suite au verso !
continuation overleaf !

69.2 N.Hadjisavvas - What a Hidden Variables Theory is not

The history of the hidden variables question is almost as old as the history of Quantum Mechanics. During this long period, many precise mathematical definitions of what is meant by the term "hidden variables theory" (HVT) were given. However, none of these definitions encountered a universal acceptance. This state of affairs becomes quite clear by a comparison of "impossibility proofs" versus concrete constructions of HVT.

On the other hand, between the variety of these definitions there exists one, who seems to be accepted by most physicists. We shall refer to it as SD (S for standard). According to SD, in order to establish a HVT, one has to specify:

a) A probabilisable space $(\Sigma,\mathcal{T})$, i.e. a set Σ, interpreted as the set of hidden variables, and a σ-algebra of subsets of Σ ($\mathcal{T}$ is necessary for the definition of probability measures).

b) For each quantal state W, a probability measure μ_W on $(\Sigma,\mathcal{T})$. This measure is thought to express the probability distribution of hidden variables in a statistical ensemble of quantum systems which is described as a whole by the state W.

c) For each observable A, a random variable f_A on $(\Sigma,\mathcal{T})$. Thus, if the intrinsic state of an individual system is $\lambda \in \Sigma$, a measurement of the observable A will yield the value $f_A(\lambda)$.

Furthermore, compatibility with Quantum Theory is ensured by the following condition:

d) The quantum-mechanical expectation value $E_W(A)$ of the observable A in the quantal state W is given by the expectation value of the random variable $f_A(\lambda)$ for the probability measure μ_W, i.e.:

$$E_W(A) = \int_{\lambda \in \Sigma} f_A(\lambda)\, d\mu_W(\lambda)$$

The above definition, with some slight variants, is very frequent in the literature of HVT {1-3}. However, this definition is naive and inconvenient, for it is at the same time too restrictive, excluding a large class of interesting HVT, and too wide, permitting theories which by no means can be considered as HVT. The scope of this letter is to explain this fact.

That the definition is too restrictive can be easily seen. Indeed, by condition (c) above, the measured value of an observable should depend solely on the hidden state of the system: In particular, the hidden variables characterising the measuring device should have no effect on the determination of this value. However, the study of all known "impossibility proofs", as well as some common sense, suffices to show that this restriction must be dropped. An extensive analysis of how condition (c) should be, can be found in Ref {7}.

On the other hand, the fact that SD is also too wide, and thus additional restrictions should be imposed, has escaped attention of many physicists. That is why, even if it is recognized that the hidden variables of the measuring devices could play a role, it is usually accepted that in any case, if one succeeds to construct a theory obeying to the conditions of SD, so much the better! This theory can be certainly thought as a HVT {1-3,5,6}.

Nevertheless, it is easy to specify constructions satisfying to the conditions of SD, and which cannot be considered as establishing a HVT. In fact, Quantum Mechanics, as it stands, can be formulated in a way to satisfy the condition of SD! Let us give two examples:

Example I

a) Take Σ to be the set of all statistical operators and $\mathcal{T}$ the set of all subsets of Σ.

b) For any quantal state W, define μ_W as the point measure concentrated on W, i.e.: $\mu_W(\{W\}) = 1$, $\mu_W(\Sigma \setminus \{W\}) = 0$.

c) For any observable A, represented by the self-adjoint operator $\hat{A}$, define the random variable:

$$\forall W' \in \Sigma : f_A(W') = \mathrm{Tr}(AW').$$

It is then easy to verify that condition (1) holds:

$$E_W(A) = \mathrm{Tr}(AW) = \int_{W' \in \Sigma} d\mu_W(W') \cdot f_A(W')$$

Example II

a) Take Σ to be the Hilbert space H representing the system, $\mathcal{T}$ the set of all subsets of H.

b) For any quantal state represented by the statistical operator W, define the measure μ_W on $(\Sigma, \mathcal{T})$ as follows: If $W = \sum_i \alpha_i P_{[\varphi_i]}$ is the spectral decomposition of W ($[\varphi_i]$ being the ray containing the normalised eigenvector φ_i, and $P_{[\varphi_i]}$ the projector on φ_i), then μ_W will be the discrete measure concentrated on the vectors φ_i, and such that $\mu(\varphi_i) = \alpha_i$ (if W has degenerate eigenvalues, one shall have to choose and fix the φ_i).

c) For any observable A represented by the self-adjoint operator $\hat{A}$, set

$$f_A(\varphi) = (\varphi, \hat{A}\varphi).$$

d) It is then easy to see that (1) holds:

$$E_W(A) = \sum_i \alpha_i (\varphi_i, \hat{A}\varphi_i) = \int_{\varphi \in H} d\mu_W(\varphi)\, f_A(\varphi)\ . -$$

It may seem incredible, and yet a great number of publications propose "hidden variable theories" which are, in fact, variants of the examples given above (cf. {1,2,5, 6,...} and the recent book of A.S.Holevo {10}, Ch.I.7).

The inadequacy of SD is thus proved. But then a new question arises: How to define properly the term "hidden variable theory"? We already saw that condition (c) of SD should be relaxed. But in view of the examples given above, it is evident that in the same time, some supplementary conditions should be imposed. There exists such a condition, the necessity of which is evident, namely, that the random variable f_A associated to an observable A, should take on values $f_A(\lambda)$ belonging to the spectrum of A and, in addition, the HVT should reproduce completely the probability measure corresponding to A, and not only mean values as in relation (1). This condition rules out examples I, II (and also a "hidden variable theory" studied in Ref.

{1,2,6,8})*. However it should be emphasized that even this enforced condition is not sufficient to arrange matters: Indeed, Kochen and Specker constructed an example which obeys to this condition also and, just like our examples I and II, has nothing to do with HVT {9}.

As a conclusion, we could say that in spite of the fact that the various "impossibility" or "possibility" proofs helped to clarify what can be expected from a HVT to do, the statement of the precise conditions which a theory should satisfy in order to be considered as establishing a hidden variable representation of Quantum Mechanics, still remains an important, unsolved problem.

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*) It follows that this condition is stronger than condition (1) and not equivalent, as erroneously stated in reference {3}, p.262.