Let  $f: \mathbb{R} \to \mathbb{R}$  be a bijection (i.e., 1-1 and onto), such that

$$\forall x \in \mathbb{R}, f(x) + f^{-1}(x) = x.$$

It is a standard exercise, found in some textbooks, to show that:

(i) f is odd.

Let me add two additional questions to this:

- (ii) Show that f is discontinuous.
- (iii) Examine whether such a function exists.

back