

Constantin Carathéodory: His life and work

By Christine Phili
National Technical University of Athens

I. An International journey

A greek of the diaspora – descendant from a family of erudites, phizicians and diplomats, well – educated, with many intellectual inferents – Constantin Carathéodory was born on 13 September of 1873 in Berlin where his father Stephanos represented the Ottoman Empire in Berlin’s Conference (arrangement of its frontiers). Two years later S. C. was appointed plenipontary ambassador in Brusseles – a post which he maintained until his retirement (1902) and the capital where he lived the rest of his life (1907).

C. C. lost his mother when he was 6 years old and he was brought by his grandmother Euthalia Petrokokinnou (descendant of a noble family of Chio’s island) in a house frequented by diplomats (Graf Brandenburg, Lord Vivian), by musicians (Massenet, Paderewski), by historians of music (Gevaert, Bourgault – Ducondray), by painters (Paul Meyerheim, Emile Wanters, Constantin Meunier), as also by professors of the International Law Aphonze Rivier, F. Martens. Perfectly biliugual in Greek – French (later he learnt German, Italian, English, Turkish and Dutch). He started his winter of 1881. He went to the private primary school of Vanderstock and later for reasons of health he attended a primary school in the Italian Riviera (1883 – 85) (in Bordighera and San Reno). In the classical high school in Brusseles appeared his love for geometry. In 1886 he studied in the gymnasium Athénée Royal d’Ixelles, from where he graduated in 1891. Although he was qualified for classical studies (later when he was professor in Göttingen, he read many Greek and Latin authors) he was attracted by the quality of the teaching of mathematics by prof. Angenot. For two successive years, he won the first prize in mathematics in the Concours Généraux.

The famous Ecole Polytechnique in Paris – created by the French Revolution and center (together with the Ecole Normale Supérieure) of the higher mathematical education in France – became a model for establishing other analogous institutions in Europe. Following the traces of the distinguished school, the French officer J. Chapelié (1792-1864) established in 1834, in Belgium the Military School of Brusseles and became its first commander. The presence of the geometers Dandelin (1794-1847), Quetelet (1796-1874). Stressed the primary role of mathematics.

So when in 1891 C. C. entered this Institution mathematics constituted and important part of his formation. As he remarked – in his Autobiographical Notes – he studied Infinitesimal Calculus from the classical book of Ch. Sturm, while the teaching of the descriptive geometry by chomé, - who followed Mongian style – familiarized him with geometry. The exemplary teaching of Ch. Lagrange in Probabilities, Astronomy and geodery completed his mathematical education.

In 1895 the young engineer paid a visit to Alexander S. Carathéodory in Canea (his relative who later became his father in law) commander of Crete, where he met for the first time Eleftherios Venizelos (1864 – 1936), the most important statesman of Greece in the XXth cenutry. During his stay he established a strong friendship with

Venizelos and probably the discussions with the expert diplomat and amateur of mathematics modified the plans of the young engineer.

It seems that the events of 1895, forced him to abandon Crete and to take refuge in Mytilene where his cousin Ioannis Aristarchis constructed the road network of the island. Coustantin helped him to sketch the roads of the island of Samos, but the greek-turkish war of 1897 prevented its realization.

After short stays in London and Paris, he started in the winter of 1858 to work as assistant engineer in the construction of the barrages of Assouan and Asiout. There, after this hard work he could, as mentioned in his Autobiographical Notes, study pure Mathematics from the books of C. Jordan, *Cocers d' Analyse* (2nd ed. and the *Analytic Geometry of Salman-Fiedler*.

In the beginning of 1900, he abandoned Egypt and his career as engineer. His book *an Egypt* (Athens 1901) constitutes his farewell gift to this fascinating country. He took the decision against the objections of his family and those of his friends, to devote himself to mathematics. Hesitating between Paris and Germany he chose Germany and in the first days of May, he registered in the University of Berlin, where the brilliant trade of Kronecher, Kummer, Weierstrass was succeed by the promising one of Fuchs, Schwarz, Frobenius.

With enthusiasm he attended their lectures, but he became friends only with H. A. Schwarz- who was professor in the Polytechnical School of Zurich of his uncle Telemachos Carathéodory – from whom he learnt the foundations of Projective Geometry and theory of Functions. There he made two faithful friends E. Schmidt and L. Fejér.

When in 1901 Schmidt continued his studies in Göttingen, Carathéodory following his friend's advise moved in Göttingen too, where F. Klein and D. Hilbert taught. With Klein – who he later succeed – he discussed mathematics, the French geometrical school of Mouge – question which Caratheodory knew well from the period of Belgiau Military School – the reform of curriculum in high schools, while attending the course of the History of Mathematics which Klein taught, continued to teach it privately (at home) to the young gifted mathematician.

When in 1903 Hans Hahn came from Vienna to Göttingen, he gave a lecture in the Mathematical Society of Göttingen on Variations Calculus, especially on the theory of Escherich of two variations in Lagrange's problem. This lecture opened new horizons to the young student, who wishing to construct a geometrical example, he conceived the subject of his Thesis. His collaboration with Schluvarz in Berlin on E- Weierstrassian work "On the discontinuous solutions in Variations Calculus", which putting it in the hands of Minkowski (because he felt him more familiar than Hlilbert and Klein), he defended it in July of 1904. Few weeks later, attending the 3rd International Congress of Mathematicians in Heidelberg, he had the opportunity to met some great mathematicians as Painlevé, Lorentz, Lindolóf.

Although his thesis offered many possibilities for a distinguished academic career, it couldn't open the gates of the University of Athens or those of the Military School as in this period there didn't exist any available post. So following Hilbert's advise he started to work on his Habilitationsschrift. Strong meanima and minima in simple integrals, which he presented in 1905 and he was nominated Privatdozent in the University of Göttingen.

As Privatdozent of Göttingen University (he preserved this post until 1908) he started to teach cinematics. At the end of semester, P. Bourtroux arrived in Göttingen and Carathéodory's discussions with him on Borel's proof on Picard's theorem, appeared in his communication in the French Academy of Sciences (1905) where he generalized Picard's theorem.

In 1908 he was elected ordinary professor in Hannover's Technical University. This same year he presented in *Mathematische Annalen* his paper "Research on the Foundation of Thermodynamics", which as it was published in a mathematical journal was later discovered by the physicists. Only when in 1921 Max Born referred to it won the interest and the praise of Max Planck and A. Sommerfeld. In this paper he exposes an alternative logical structure of the foundation of Thermodynamics (see Carathéodory's Principle). The next year he became ordinary professor in the newly established Technical University of Breslau, while the young Cretian lawyer Venizelos, has been elevate Greece's leader and desiring to organize the Universitarian Community invited him to participate in the electoral jury for professorship in Athens University.

The next years C. C. succeeded (1913) Klein in the University of Göttingen, while in 1918 he became Frobenius' successor Frobenius in the University of Berlin, a post which he maintained for two years approximatively, when the Greek Prime Minister Eleftherios Venizelos invited him to contribute activitely to an enterprising and ambitious plan: the Foundation of the University of Smyrni.

II. The University of Smyrni - Light from the East

With his book on Theory of function (Vorlesungen über reelle Funktionen Teubner 1918) he established his international reputation and the next year 1919, the father of Quantum theory, Max Planck welcomed him with an inspiring speech in the Prussian Academy of Sciences.

The reestablishment of the Ionian coasts as a beacon of Hellenism constitutes one of the basic axes Venizelos' policy. From the beginning of 1919 the idea to create a University dominated his thoughts, and Carathéodory adopted it and participated in its realization. On 20 October of 1919 he submitted his note on the University's function in Smyrni (liberated from 2.5.1919) having as a model the Anglo – Saxon Universities. In 1920 he abandoned an international career to support the dream of his old friend. Two months later on the cruiser Elli in the bay of Smyrni, Venizelos, Carathéodory and some others studied the function of the new University. Carathéodory wished to bring together all the Greek personalities to reach their target. So he invited the professor of the University of Berlin G. Ioakimoglou – born in Ionia- to assist in its organization. Ioakimoglou adopting the exhortation of the head of the nation, that Greece must civilize Minor Asia, accepted the invitation.

By a decree of the High Committee, the University of Smyrni was founded, having Greek as its official language and comprised the following departments:

1. School of agriculture (to train botanologists, zoologists and to organize courses for Peasant) and of sciences (to train civil engineers, architects, chemists, geologists etc).
2. School of eastern languages (Turkish, Arabic, Persian, Armenian ancient and new hebrew).
3. School for the staff of the state.
4. Commercial school.
5. School of leveling and foreman.
6. High Islamic theological seminar.
7. Institute of sanitation (with free tests, preparation of vaccines etc).
8. Foundation of Public Library.

Carathéodory tried to create a rich library unique in the whole of the East as also to provide instruments of the microbiological laboratory (with the assistance of Ioakimoglou). For the establishment of this library Carathéodory engaged Dr. Anserer (staff of the National Library of Berlin who speaks fluent Turkish) and he created an unofficial annexe side by side with the official office where all the information concerning the sale of private libraries and the catalogues of double reprints in public German libraries were gathered. So with the assistance of the Greek state Carathéodory could buy these books which were not needed for the University of Louvain (Germany in Versailles agreed to replace the library of the University, which was burnt in August 1914). These books packed in 36 large boxes, were sent to Smyrni. To those were added an important number of rare books which were collected by the Austrian archaeologists in Ephesos. Carathéodory having a deep culture in art and aesthetics (he was a renowned

collector), he took an interest in the University's furniture and obtained it from the well known Berliner House Zelder and Platen.

As for the staff, Carathéodory proposed (besides Prof. Ioakimoglou), for the chair of Physics, F. Theodorides, a graduate of the Polytechnic School of Zurich and student of Pierre Weiss, for the chair of chemistry, P. Karopoulos, assistant of Tamann. For the department of agriculture he indicated Th. Kessissoglou, a graduate of the agricultural School of Gembloux (Belgium) and organizer of some agricultural schools in China, Colombia and Uruguay. Also following Carathéodory's proposition, Paschevitz was appointed machinist of the University; its secretary was Nicolaos Kriticos (who later became Professor of the National Technical University of Athens) who in this period served in the (army) division of Cydonies (Asia Minor).

In October of 1921 Carathéodory returned to Smyrni and worked with enthusiasm for the organization of the University, whose emblem was defined by him: Light from the East, believing that the new Institution will be the most perfect University of the East. The lectures never started as the tragic events of August 1922 swept anything. His daughter Mrs Despina Rodopoulou – Carathéodory referred to this period: "He stayed to save anything he could: library, machines etc which were shipped in different ships hopping that one day they will arrive in Athens. My father stayed until the last moment. George Horton, consul of U.S.A. in Smyrni wrote a book ... which was translated in Greek. In this book Horton notes: "One of the last Greek I saw on the streets of Smyrna before the entry of the Turks was Professor Carathéodory, president of the doomed University. With him departed the incarnation of Greek genius of culture and civilization on Orient".

A refugee in Athens too, he was nominated Prof. of Mathematical Analyses in the University of Athens (1922) and Professor of Mechanics in the National Technical University of Athens (1923). Nevertheless in these difficult years the horizon of his scientific activity remains limited. "they make great efforts to keep me here – he wrote to his friend I. Kalitsounaki in 24.2.1924 – and of course I would wish to stay if I was sure that I could serve my country. But as is the situation in our Department (and as it continues to be for many years) it seems to me very difficult". So when in this same year (1924) the famous physicist A. Sommerfeld invite him to succeed F. Lindeman in the University of München, he accept this invitation and he stays there for the rest of his life (1950).

III. The Fertile Years

In 1928, invited by the American Mathematical Society he visited the U.S.A., as guest professor for one semester in Harvard and he gave lectures in different Universities (Austin, Berkley etc). In a letter to his relative Penelopi Delta (distinguished Greek writer, a kind of Greek Madame de Serigny), he defended the decision for his settlement abroad. *"I assure you that in the few months that I am here I achieved more than in two years in Athens. On the contrary my activity in Smyrni could be fruitful, nevertheless Greece did not lose anything from my departure, on the contrary! Because rather than waste my time there I can here be more useful through my propaganda for Greece"*.

The victory of the Liberal Party (August 1928), gave the opportunity to its founder to become concerned with the educational problem of the country. For the reform of the University education Venizelos asked for Carathéodory's contribution, which he accepted, hoping "to convince the Government to take effective measures".

So Venizelos recalled Carathéodory to reform the University of Athens and to establish the University of Thessaloniki, an idea which he had conceived at the end of the First World War. In the session of the Greek Parliament on 17 December of 1929, the Prime Minister E. Venizelos declared: "I remind you that after the end of the Great War, Venizelos' Government at that time decided not only to establish a second University but a third one ... Wishing that the new Universities will be established as perfectly as possible, it recalled Professor Carathéodory, who is not only a great mathematical genius but he is preeminently distinguished organizer of Institutions, since the Great Germany changed him the organization of the Polytechnical School of Breslau. So, we invited Mr. Carathéodory and we told him "We need to establish these two Universities, we consider you as the best one to organize them. We think that it is better to start with the University of Smyrna and after that of Thessaloniki".

In 1930 he was appointed Governmental commissioner (a post which he preserved until 1933) in the Universities of Athens and Thessaloniki. In the sessions of the University Senate (according to the law 5143 of 10.7.1931 he was Rector) he gave the directions for the creation of internal regulations, he planned the formation of the veterinarian school (unique in Greece which starts its function only in 1950). "The Reform of the University of Athens" Athens (1930) is his report which he submitted to the Government about the function of the National and Kapodistrian University, where he proposes measures for the improvement of the financial situation of the institution, the restriction of the number of students, the change of the method of exams, the way to elect the professorship etc. This report constituted the kernel of the framework of the Universities function following the law of 5343/1932 until 1982.

Wishing to contribute the Greek mathematical activities, he participated in 1934 in the First Interbalcanic Mathematical Conference with his paper "On the Equations of Mechanics" which was presented by his old colleague in Berlin University, Prof. Nicolaos I. Hadjidakis.

In 1936, visiting again the States, he was impressed by the progress of Greek Scientists, as he wrote in a letter addressed to the Rector of the University of Athens, Professor Panagiotis Zervos "what made a great impression on me in America is the truly great progress of Greek scientists ... not only the number of Greeks who we met as professors in the Universities is astonishingly great, but some among them occupy perfectly exceptional posts".

This same year, as member of the Committee for the Fields medal with cartau, Birchoff, Tagaï and president F. Severi, they decided to award it, deering the 10th International Congress in Oslo to Jesse Douglas and Lars Ahlfors.

In 1937 he returned once more to Greece and gave lectures in the National Technical University and he participated in the festivities for the centenary of the Archeological Society with his paper "on the curves of the pedestral of the Parthenon and on the distance of its columns", where he rejected the conception, of Penrose and Stevens who asserted that the curves of the Parthenon are paraboles. Carathéodory remarks that

Iktinos constructed the only curves known in his period, circles of great diameters, since the concept of conic sections came after the 5th century.

IV. His contribution to Mathematics.

For many years Carathéodory was the editor of *Mathematische Annalen* and member of the editorial board of *Circolo Matematico di Palermo* [earlier another distinguished Greek mathematician was on the editorial board of this same journal, KyparissosStephanos (1857 – 1917)]. He was elected member of many Academies (Berlin, München, Göttingen, Bologna, Lincei, Athens), while noteworthy is his election in the Papal Academy.

“Every thing which causes our admiration, always according to Euclid’s recollection, is the unified perfection of two abilities, which are rarely found together in the human intellect. That of the imaginative surveillance of space together with the deepest power of the abstract calculus. This combination resulting from the inherited treasure of distant ancestors, constitutes the philosophical stone of Carathéodory’s creation”. With these words, E. Schmidt describes the background of the man who could be compared with the ancient Greeks, in his post mortan tribute to Carathéodory (Ges. W. Bd v p. 412).

His research covers many fields of mathematics (pure and applied). His chance entry (as that of the young Lagrange few centuries ago) in to the temple of mathematics is marked by his interest in the calculus of variations, on which even later he continued to work. Previously, in first degree problems of variation it was only known as Erdmann corner condition and Carathéodory in his Thesis (1904) demonstrated that the theory of smooth functions could be extended to curves with corners. Besides this, as he knew quite well the history of this branch [later he wrote the “Introduction into Euler’s publications on the calculus of variations in L. Euler Opera Omnia (1) 24 1952 p. viii-lxii] he generalized an idea of Johann I Bernoulli in the resolution of the brachistochrone problem which became a starting point.

The theory of functions constitutes Carathéodory’s second main field of research which extended in many areas. His first communication “On some generalizations of M. Picard’s theorem” in the French Academy of Sciences in 1905, led him to many generalizations see f. ex. his work with E. Landau “Contribution on convergence of functions sequences” (1911) or “About a generalization of Picard’s theorems” (1920), which both of them are published in the proceedings of the Prussian Academy of Sciences.

Another area comprises coefficients problems in expansions in a power series which he starts to work in 1907 see “About variation’s domain of power series coefficients which did not accept a given value” in *Mathematische Annalen*; as also Fourier’s coefficients see “on the constant Fourier’s coefficients of monotone functions” (1920) in the Proceedings of the Prussian Academy of Sciences.

Problems arising from Schwarz’s lemma (which took this name from his proposition) composes an important part of his research see f. ex. “A generalization of Schwarz’s lemma” *Bull. Amer. Math. Soc.* (1937).

He makes another significant advance in the theory of several variables see f. ex. “A theorem for analytic functions of several variables analogous to Vitali’s theorem” in Crelle’s Journal (1931).

Nevertheless his remarkable contribution is focused in the field of conformal representation on which he started to work in 1912-1913. “Research on conformal representation in constant and variable regions”. (Math. Ann. 1912) or “About the reciprocal correspondence of boundary in conformal representation in the inner of Jordan’s curve”. Math. An. 1913.

After Riemann’s first form of conformal representation in 1851, the main theorem of conformal representative of simply connected regions on the circle of unit radius, was demonstrated (in a greatly simplified proof) by Carathéodory (see “Elementary Proof of the fundamental theorem of conformal representation” published in the Math. Abh. For H. A. Schwarz’s 50th anniversary of his thesis (1914). Nevertheless his main contribution is his theory of boundary correspondence in which he introduced the geometrical set theoretic properties of those boundaries (theory of *Primenolen*).

His third main field comprises the theory of real functions. The profound study of C. Jordan’s book *Cours d’Analyse* [2nd ed. (1893) where the author presented the new analysis as it was developed by the works of Weierstrass, Dedekind, Cantor and Jordan himself] offers him the rigorous background for his research which took its crystallized form in his book *Lectures on real functions* (1918) where he completes Borel and Lebesgue and makes his first step of axiomatization. O. Perron’s in his tribute for Carathéodory, stresses that with this book Carathéodory gave a wonderful gift to the mathematicians. He also worked on the theory of measure of point – sets and of the integral. His paper “On the linear measure of point sets, a generalization of length’s concept” in the Scientific Society of Göttingen (1914) is characteristic. Carathéodory returned to it, in a more generalized and abstract form, which he called an algebrization. “Algebrization’s sketch of Integral’s concept” which he presented in 1938 in the Proceedings of the Bavarian Academy of Sciences.

Wishing to reform his classic book on real functions, decided that its 3rd edition will be divided it in volumes the first one comprises Numbers, Point – sets, Functions and in the second volume, as his student Nicolaos Criticos (1894-1985) pointed out in his obituary, would comprise the new Carathéodory’s theory concerning our objects of thought which the great mathematicians names them *Soma* in German (from the Greek word *σώμα* =body). These *Somen* are defined by simple postulates and so the well known concepts of Lebesgue’s and Stieltjes integrals result as an application of his theory. Probably Hilbert’s *Grundlagen der Geometrie* (1899) and his conviction that the objects of mathematical thought are the symbols themselves (Math. An. 1926) inspired him to this attempt of abstractness. The form of his theory appeared in his posthumous book *Measure and Integral and its algebrization* (1956).

Ending our tribute to C.C., we wish to stress his interest in applied mathematics. His papers on thermodynamics are noteworthy. The first *one Research on the Foundations of thermodynamics* (Math. An. 1909) was complete in 1925 by another important paper: *On the determination of the energy and the absolute temperature with unreversible process*, which was published in the Proceedings of the Berlin Academy of Sciences. His interest in the theory of relativity was expressed in 1924 in his paper: *On the axiomatization of the special theory of relativity* presented in the Proceedings of the

Prussian Academy of Sciences where he exposes the axiomatic foundation of the Einstein's special theory of relativity where he presented the general equations of transformations partial case of them are Lorentz – Minkowski' transformations.

As Perron pointed out C.C. was very interested in the History of Mathematics. His knowledge of ancient Greek mathematics was remarkable. His friend, professor I. Kallitsounakis in his post humorous tribute to Carathéodory in the Proceedings of the Academy of Athens (session of 23rd February 1950), reveals that in the last years of his life he encouraged him to work on Plato's mathematics. But Carathéodory considered that his age constituted an obstacle to working in this vast area of research.

Nevertheless his knowledge of the history of mathematics not restricted only to ancient Greek mathematics. He knew quite well Euler's work as that of his contemporaries. He wished to understand the mathematical methods of every period and from the precious Kernel of the past to develop the exact solution of present problems. So apart from his contribution on Euler's Opera Omnia he presented various papers as f. ex. *The beginning of Research in the calculus of Variations*. Osiris (1937) or *the Significance of Erlanger Program* (1919) or his contribution on *Mathematics* (with W. van Dyck) for the 50 years of German Science. For the 70th anniversary of Friedrich Schmidt – ott.

“All the publications of Carathéodory – wrote Nicolaos Criticos – have some – thing plastic and all are well crafted, even the less important, with the love and persistence and conscientiousness of a good craftsman, thus giving the lecture an analogous aesthetic pleasure, like the well conceived and with perfection presented an work of art”.

Bibliography

Carathéodory's articles were collected in *Cesammelte Mathematische Schriften* in 5 Vols (Munich 1954 – 1957) (in 1944 abroad the Bavarian Academy of Sciences decided to edit Carathéodory's *Opera Omnia*).

Carathéodory's books are:

Vorlesungen über reelle Funktionen

Teubner, Leipzig – Berlin 1918; 2nd edition 1928).

Conformal Representation

Cambridge Tracts in Mathematics and Mathematical Physics No 28 (Cambridge, 1932).

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Mars und Integral und ihre Algebraisierung, Birkhäuser, Basel 1956.

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