CONSTANTIN CARATHÉODORY: HIS LIFE AND WORK

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I. AN INTERNATIONAL JOURNEY

A Greek of the diaspora - descendant from a family of erudites, physicians and diplomats, well-educated, with many intellectual interests - Constantin Carathéodory was born on 13 September of 1873 in Berlin where his father Stephanos represented the Ottoman Empire in Berlin's Conference (the Conference treated the arrangement of the Empire's frontiers). Two years later S. C. was appointed plenipotentiary ambassador in Brussels, a capital where he lived even after his retirement (1902) until the end of his life (1907).

C. C. lost his mother when he was 6 years old. He was brought up by his grandmother Euthalia Petrokokinnou (descendant of a noble family of Chio's island) in a house frequented by diplomats (Graf Brandenburg, Lord Vivian), by musicians (Massenet, Paderewski), by historians of music (Gevaert, Bourgault - Ducoudray), by painters (Paul Meyerheim, Emile Wanters, Constantin Meunier), and also by professors of international Law (Aphonse Rivier, F. Martens). Perfectly bilingual in Greek and French (later he learnt German, Italian, English, Turkish and Dutch), he started his education in the winter of 1881. He went to the private primary school of Vanderstock, and later for reasons of health he attended a primary school in the Italian Riviera (1883 - 85, in Bordighera and San Remo). In the classical high school in Brussels he was especially attracted to geometry. In 1886 he studied in the gymnasium Athénée Royal d'Ixelles, from where he graduated in 1891. Although he was qual-

ified for classical studies (later when he was professor in Göttingen, he read many Greek and Latin authors) he was attracted by the quality of the teaching of mathematics by Professor Angenot. For two successive years, he won the first prize in mathematics in the Concours Généraux.

The famous Ecole Polytechnique in Paris, created by the French Revolution and (together with the Ecole Normale Supérieure) center of the higher mathematical education in France, became a model for establishing other analogous institutions in Europe. Following the traces of the distinguished school, the French officer J. Chapelié (1792-1864) established in 1834 in Belgium the Military School of Brussels and became its first commander. The presence of the geometers Dandelin (1794-1847) and Quetelet (1796-1874) stressed the primary role of mathematics.

So when in 1891 C. C. entered this Institution, mathematics constituted an important part of his formation. As he remarked in his Autobiographical Notes, he studied Infinitesimal Calculus from the classical book of Ch. Sturm, while the teaching of the descriptive geometry by Chomé, - who followed Mongian style - familiarized him with geometry. The exemplary teaching of Ch. Lagrange in probabilities, astronomy and geodesy completed his mathematical education.

In 1895 the young engineer paid a visit to the commander of Crete Alexander S. Carathéodory (his relative who was to become his father in law), where he met for the first time Eleftherios Venizelos (1864 - 1936), the most important statesman of Greece in the XX*th* century. During his stay he established a strong friendship with Venizelos and probably the discussions with the expert diplomat and amateur of mathematics A. Carathéodory modified the plans of the young engineer.

It seems that the events of 1895 forced him to abandon Crete and to take refuge in Mytilene, where his cousin Ioannis Aristarchis constructed the road network of the island. Constantin helped him to plan the roads of the island of Samos, but the Greek-Turkish war of 1897 prevented its realization.

After short stays in London and Paris, in the winter of 1858 he started to work as assistant engineer in the construction of the barrages of Assouan and Assiout. There, after working hard all day (as mentioned in his Autobiographical Notes), he studied pure mathematics from the books of C. Jordan, Cours d'Analyse (2nd ed.) and the Analytic Geometry of Salmon-Fiendler.

In the beginning of 1900, he left Egypt and abandoned his career as engineer. His book on Egypt (Athens 1901) constitutes his farewell gift to this fascinating country. He took the decision, against the objections of his family and those of his friends, to devote himself to mathematics. Hesitating between France and Germany he chose Germany and in the

first days of May, he was registered in the University of Berlin, where the brilliant triad of Kronecker, Kummer and Weierstrass was succeeded by the promising one of Fuchs, Schwarz and Frobenius.

He attended their lectures with enthusiasm, but he befriended only H. A. Schwarz, who was professor of his uncle Telemachos Carathodory in the Polytechnical School of Zurich, from whom he learnt the foundations of Projective Geometry and theory of Functions. There he made two other faithful friends: E. Schmidt and L. Fejér.

When in 1901 Schmidt continued his studies in Göttingen, Carathéodory following his friend's advise moved to Göttingen too, where F. Klein and D. Hilbert taught. With Klein - whom he later succeeded - he discussed on mathematics, on the French geometrical school of Monge (a subject that Carathéodory knew well from the period of Belgian Military School) and the reform of curriculum in high schools, while attending the course of the History of Mathematics which Klein taught in the University. Later Klein continued to teach it privately (at home) to the young gifted mathematician.

When in 1903 Hans Hahn came from Vienna to Göttingen, he gave a lecture in the Mathematical Society of Göttingen on the Calculus of Variations, especially on the theory of Escherich of two variations in Lagrange's problem. This lecture opened new horizons to the young student who, wishing to construct a geometrical example, conceived the subject of his Thesis. His collaboration with Schwarz in Berlin on E-Weierstrassian functions, led to the final form of his work "On the discontinuous solutions in the Calculus of Variations". He gave it first to Minkowski (because he felt closer to him than to Hilbert and Klein), and defended it in July of 1904. A few weeks later, attending the 3rd International Congress of Mathematicians in Heidelberg, he had the opportunity to meet some great mathematicians such as Painlevé, Lorentz and Lindolöf.

Although his thesis offered many possibilities for a distinguished academic career, it could not open the gates of the University of Athens or those of the Military School as in this period no position was available. So following Hilbert's advice he started to work on his *Habilitationsschrift* "Strong maxima and minima in simple integrals", which he presented in 1905. With it he was nominated Privatdozent in the University of Göttingen.

As Privatdozent of Göttingen University (a post he occupied until 1908) he started to teach kinematics. At the end of the semester, P. Bourtroux arrived in Göttingen. Carathéodory's discussions with him on Borel's proof of Picard's Theorem, led to a generalization of the

theorem that appeared in his communication in the French Academy of Sciences (1905).

In 1908 he was elected professor in Hannover's Technical University. This same year he presented in Mathematische Annalen his paper Research on the Foundation of Thermodynamics, which, being published in a mathematical journal, was initially unnoticed by physicists. As late as 1921, after Max Born referred to it, C.C. won the interest and the praise of Max Planck and A. Sommerfeld. In this paper he exposes an alternative logical structure on the foundation of Thermodynamics (the so-called Carathéodory's Principle). The next year he became full professor in the newly established Technical University of Breslau. At this time the young Cretan lawyer Venizelos, already elevated to Greece's leadership and desiring to organize the University Community, invited him to participate in the evaluating committee for professorships in Athens University.

The next years C. C. succeeded Klein in the University of Göttingen, (1913) while in 1918 he became Frobenius' successor in the University of Berlin, a position he maintained for two almost years, until the Greek Prime Minister Eleftherios Venizelos invited him to participate actively to an enterprising and ambitious plan: the Foundation of the University of Smyrni.

II. THE UNIVERSITY OF SMYRNI - LIGHT FROM THE EAST

With his book on the Theory of Functions (Vorlesungen ber reelle Funktionen, Teubner 1918) C.C. established his international reputation. Next year 1919 Max Planck, the father of Quantum theory, received him with an inspiring speech in the Prussian Academy of Sciences.

The reestablishment of the Ionian coasts as a beacon of Hellenism constitutes one of the basic axes of Venizelos' policy. From the beginning of 1919 the idea to create a University dominated his thoughts, and Carathéodory adopted it and participated in its realization. On the 20th of October 1919, he submitted his proposal on the University's foundation in Smyrni (liberated from the Turks on 2.5.1919) having as a model the Anglo-Saxon Universities. In 1920 he abandoned an international career to support the dream of his old friend. Two months later on the cruiser Elli in the bay of Smyrni, Venizelos, Carathéodory and some others made plans for the new University. Carathéodory wished to bring together all Greek personalities to reach their target. So he invited the professor of the University of Berlin G. Ioakimoglou - born in Ionia- to

assist in its organization. Ioakimoglou adopting Venizelos' exhortation, that Greece must civilize Minor Asia, accepted the invitation.

By a decree of the High Committee, the University of Smyrni was founded, having Greek as its official language and comprising the following departments:

1. School of agriculture (to teach botanologists and zoologists, and to provide training to farmers) and of sciences (to teach civil engineers, architects, chemists, geologists etc).

2. School of eastern languages (Turkish, Arabic, Persian, Armenian, ancient and new Hebrew).

3. School of State administration.

4. Commercial school.

5. School of foremen.

6. High Islamic theological seminar.

7. Institute of sanitation (with free tests, preparation of vaccines etc).

8. Library science.

Carathéodory tried to create a rich library unique in the whole of the East and also to provide instruments for the microbiological laboratory (with the assistance of Ioacimoglou). He engaged Dr. Auserer for the library (a member of the National Library of Berlin who spoke Turkish fluently) and created an annex where all information concerning the sale of private libraries and the catalogues of double reprints in public German libraries was gathered. So with the assistance of the Greek state Carathéodory was able to buy these books from the University of Louvain (Germany in Versailles agreed to replace the library of the University, which was burnt in August 1914). The books were packed in 36 large boxes and sent to Smyrni. To those an important number of rare books were added, collected by the Austrian archaeologists working in Ephessos. Carathéodory, being highly cultivated in art and aesthetics (he was a renowned collector), took an interest at the University's furniture and obtained it from the well-known Berliner House Zelder und Platen.

As for the faculty, Carathéodory proposed (besides Prof. Ioakimoglou) F. Theodorides (a graduate of the Polytechnic School of Zurich and student of Pierre Weiss) for the chair of Physics, and P. Karopoulos (assistant of Tamann) for the chair of chemistry. For the department of

agriculture he indicated Th. Kessissoglou, a graduate of the agricultural School of Gembloux (Belgium) and organizer of some agricultural schools in China, Colombia and Uruguay. Also following Carathéodory's proposition, Paschevitz was appointed machinist of the University; its secretary was Nicolaos Kriticos (who later became Professor of the National Technical University of Athens) who in this period served in the army division of Cydonies (Asia Minor).

In October of 1921 Carathéodory returned to Smyrni and worked with enthusiasm to organize the University, whose emblem was defined by him: Light from the East, believing that the new Institution will be the most perfect University of the East. The lectures never started as the tragic events in August 1922 swept everything. His daughter Mrs Despina Rodopoulou-Carathéodory referred to this period: "He stayed to save anything he could: books, instruments etc. These were put in different ships hoping that one day they will arrive in Athens. My father stayed there until the very last moment". George Horton, consul of U.S.A. in Smyrni wrote a book, *All about Turkey*, which was translated in Greek. In this book Horton notes: "One of the last Greeks I saw on the streets of Smyrna before the entry of the Turks was Professor Carathéodory, president of the doomed University. With him departed the incarnation of Greek genius of culture and civilization in the Orient".

A refugee in Athens too, he was nominated Professor of Mathematical Analysis in the University of Athens (1922) and Professor of Mechanics in the National Technical University of Athens (1923). Nevertheless, in these difficult years the horizon of his scientific activity remains limited. "*They make great efforts to keep me here* – he wrote to his friend I. Kalitsounakis in 24.2.1924 – *and of course I would wish to stay if I was sure that I could serve my country. But given the situation in our Department (as it continues to be for many years) this seems to me to be very difficult*". So when in this same year (1924) the famous physicist A. Sommerfeld invited him to succeed F. Lindeman in the University of München, he accepted this invitation and he stayed there for the rest of his life (1950).

III. THE FERTILE YEARS

In 1928, invited by the American Mathematical Society he visited the U.S.A., as guest professor for one semester in Harvard, giving lectures in various Universities (Austin, Berkley etc). In a letter to his relative Penelopi Delta (distinguished Greek writer, a kind of Greek Madame de Sevigny), he defended the decision for his settlement abroad. "*I assure you that in the few months that I am here I achieved more than in*

two years in Athens. My activity in Smyrni could be fruitful, nevertheless Greece did not lose anything from my departure, on the contrary! Rather than waste my time there, I can be here more useful through my propaganda for Greece".

The victory of the Liberal Party (August 1928), gave the opportunity to its founder to become concerned again with the educational problem of the country. For the reform of the University education Venizelos asked for Carathéodory's contribution, which he accepted, hoping to convince the Government to take effective measures.

So Venizelos recalled Carathéodory to reform the University of Athens and to establish the University of Thessaloniki, an idea that he had conceived at the end of the First World War. In the session of the Greek Parliament on 17 December of 1929, the Prime Minister E. Venizelos declared: "*I remind you that after the end of the Great War, Venizelos' Government at that time decided not only to establish a second University but a third one Wishing that the new Universities will be established as perfectly as possible, it recalled Professor Carathéodory, who is not only a great mathematical genius but he is preeminently distinguished organizer of Institutions, since the Great Germany charged him with the organization of the Polytechnic School of Breslau. So, we invited Mr. Carathéodory and we told him 'We need to establish these two Universities, we consider you as the best one to organize them. We think that it is better to start with the University of Smyrni and after that of Thessaloniki'* ".

In 1930 he was appointed Governmental commissioner (a post which he kept until 1933) in the Universities of Athens and Thessaloniki. In the sessions of the University Senate (according to the law 5143 of 10.7.1931 he was Rector) he gave the directions for the creation of internal regulations, he planned the formation of the veterinarian school (unique in Greece which starts functioning only in 1950). "*The Reform of the University of Athens*" Athens (1930) is his report which he submitted to the Government about the restructuring of the National and Kapodistrian University, where he proposes measures for the improvement of the financial situation of the institution, the reduction of the number of students, the reform of the exams, the procedure for choosing professors etc. The State Law no. 5343/1932, regulating all University matters until 1982, was based on that report.

Wishing to contribute to the Greek mathematical activities, he participated in 1934 in the First Interbalcanic Mathematical Conference with his paper "*On the Equations of Mechanics*" which was presented by his old colleague in Berlin University, Prof. Nicolaos I. Hadjidakis.

In 1936, visiting again United States, he was impressed by the progress of Greek scientists, as he wrote in a letter addressed to the Rector of the University of Athens, Professor Panagiotis Zervos: "*What made a great impression on me in America is the truly great progress of Greek scientists not only the number of Greeks who we met as professors in the Universities is astonishingly great, but some among them occupy quite exceptional positions*".

This same year, he was member of the Committee for the Fields Medal with Cartan, Birchoff, Tagaki and president F. Severi. The Medal was awarded, during the 10th International Congress in Oslo, to Jesse Douglas and Lars Ahlfors.

In 1937 he returned once more to Greece and gave lectures in the National Technical University. He also participated in the festivities for the centenary of the Archeological Society with his paper "*On the curves of the pedestal of the Parthenon and on the distance of its columns*", where he rejected the conception of Penrose and Stevens, who asserted that the curves of the Parthenon are parabolas. Carathéodory remarks that Iktinos constructed the only curves known in his period, circles of great diameters, since the concept of conic sections came after the 5th century.

IV. HIS CONTRIBUTION TO MATHEMATICS

For many years Carathéodory was the editor of Mathematische Annalen and member of the editorial board of the Rendiconti del Circolo Matematico di Palermo [earlier, another distinguished Greek mathematician was on the editorial board of this same journal, Kyparissos Stephanos (1857 - 1917)]. He was elected member of many Academies (Berlin, München, Göttingen, Bologna, Lincei, Athens), while noteworthy is his election in the Papal Academy.

"*Everything which causes our admiration, always according to Euclid's recollection, is the unified perfection of two abilities, which are rarely found together in the human intellect. That of the imaginative perception of space together with the deepest power of the abstract calculus. This combination resulting from the inherited treasure of distant ancestors constitutes the philosophical stone of Carathéodory's creation*". With these words, E. Schmidt describes the background of the man who could be compared with the ancient Greeks, in his post mortem tribute to Carathéodory (Ges. W. Bd. V p. 412).

His research covers many fields of mathematics, pure and applied. His entry by chance (as that of the young Lagrange few centuries ago) into

the temple of mathematics is marked by his interest in the Calculus of Variations, on which he continued to work for a long time. Carathéodory in his Thesis (1904) demonstrated that the theory of smooth functions could be extended to non-differentiable curves. Besides this, as he knew quite well the history of this branch [later he wrote the "*Introduction into Euler's publications on the Calculus of Variations*" in L. Euler Opera Omnia (1) 24 1952 p. viii-lxii] he generalized an idea of Johann I. Bernoulli for the resolution of the brachystochrone problem which became a turning point of the theory.

The theory of functions constitutes Carathéodory's second main field of research, which he extended in many areas. His first communication "*On some generalizations of M. Picard's theorem*" in the French Academy of Sciences in 1905, led him to many generalizations as for example his work with E. Landau "*Contribution on convergence of functions sequences*" (1911) or "*About a generalization of Picard's theorems*" (1920), both of them published in the Proceedings of the Prussian Academy of Sciences.

Another area comprises coefficient problems in power series expansions, which he starts to work in 1907 (see "*On the domain of variation of power series coefficients which do not accept a given value*" in Mathematische Annalen; also for Fourier's coefficients see "*On the constant Fourier's coefficients of monotone functions*" (1920) in the Proceedings of the Prussian Academy of Sciences).

Problems arising from Schwarz's lemma (which took this name from his proposition) compose an important part of his research; see for example "*A generalization of Schwarz's lemma*" Bull. Amer. Math. Soc. (1937).

He makes another significant advance in the theory of several variables in "*A theorem for analytic functions of several variables analogous to Vitali's theorem*" published in Crelle's Journal (1931).

Nevertheless his most remarkable contribution is focused on the field of conformal representation on which he started to work in 1912-1913: see "*Research on conformal representation in constant and variable regions*". (Math. Ann. 1912) or "*About the reciprocal correspondence of boundary in conformal representation of the interior of a Jordan's curve*" (Math. An. 1913).

After Riemann's first germ of conformal representation in 1851, the main theorem of conformal representation of simply connected regions on the circle of unit radius was demonstrated (in a greatly simplified proof) by Carathéodory (see *Elementary Proof of the fundamental theorem of conformal representation* published in the Math. Abh. for H. A. Schwarz's 50th anniversary of his thesis (1914)). Nevertheless his

main contribution is his theory of boundary correspondence in which he introduced the geometrical set theoretic properties of those boundaries (theory of Primenden).

His third main field comprises the theory of real functions. The profound study of C. Jordan's book Cours d' Analyse [2nd ed. (1893) where the author presented the new analysis as it was developed in the works of Weierstrass, Dedekind, Cantor and Jordan himself] offers him the rigorous background for his research, which took its definite form in his book *Lectures on Real Functions* (1918) where he completes Borel and Lebesgue and makes his first step of axiomatization. O. Perron in his tribute for Carathéodory, stresses that with this book Carathéodory gave a wonderful gift to the mathematicians. He also worked on the measure and integration theory. His paper *On the linear measure of point sets, a generalization of length's concept* in the Scientific Society of Göttingen (1914) is characteristic. Carathéodory returned to it, in a more generalized and abstract form, in *Algebrization's sketch of Intergral's concept* which he presented in 1938 in the Proceedings of the Bavarian Academy of Sciences. The first paper of N. Bourbaki (published in 1935 in the Proceedings of the French Academy of Sciences) was related to the theory of integration and was based on one of his theorems.

Wishing to reform his classic book on real functions, he decided that its 3rd edition would be divided to two volumes. The first one comprises numbers, point-sets, functions and in the second volume, as his student Nicolaos Criticos (1894-1985) pointed out in his obituary, would comprise the new Carathéodory's theory concerning our objects of thought which the great mathematician names them *Soma* in German (from the Greek word $\sigma\acute{\omega}\mu\alpha$=body). These *Somen* are defined by simple postulates and so the well-known concepts of Lebesgue's and Stieltjes integrals result as an application of his theory. Probably Hilbert's Grundlagen der Geometrie (1899) and his conviction that the objects of mathematical thought are the symbols themselves (Math. An. 1926) inspired him to this attempt of abstractness. The form of his theory appeared in his posthumous book *Measure and Integral and its algebrization* (1956).

Ending our tribute to C.C., we wish to stress his interest in applied mathematics. His papers on thermodynamics are noteworthy. The first one *Research on the Foundations of Thermodynamics* (Math. An. 1909) was completed in 1925 by another important paper: *On the determination of the energy and the absolute temperature with irreversible processes*, which was published in the Proceedings of the Berlin Academy of Sciences. His interest in the theory of relativity was expressed in 1924 in his paper: *On the axiomatization of the special theory of relativity* presented in the Proceedings of the Prussian Academy of Sciences where

he developed an axiomatic foundation of Einstein's special theory of relativity and presented general transformation equations, partial case of which are the Lorentz - Minkowski's transformations.

As Perron pointed out, C.C. was very interested in the History of Mathematics. His knowledge of ancient Greek mathematics was remarkable. His friend, professor I. Kallitsounakis in his posthumous tribute to Carathéodory in the Proceedings of the Academy of Athens (session of 23rd February 1950), reveals that in the last years of his life he encouraged him to work on Plato's mathematics. But Carathéodory considered that his age constituted an obstacle to working in this vast area of research.

Nevertheless his knowledge of the history of mathematics was not restricted to ancient Greek mathematics. He knew quite well Euler's work as well as that of his contemporaries. He wished to understand the mathematical methods of every period and, using the precious ideas of the past, to arrive to solutions of present problems. So apart from his contribution on Euler's Opera Omnia he presented various papers as for example *The beginning of research in the calculus of variations*, Osiris (1937) or *The Significance of Erlanger Program* (1919) or his contribution on *Mathematics* (with W. van Dyck) for the 50 years of German Science on the occasion of the 70th anniversary of Friedrich Schmidt - Ott.

"*All publications of Carathéodory* – wrote Nicolaos Criticos – *have something plastic and all are well crafted, even the less important, with the love and persistence and conscientiousness of a good craftsman, thus giving to the lecture an analogous aesthetic pleasure, like the well-conceived and presented with perfection work of art*".

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