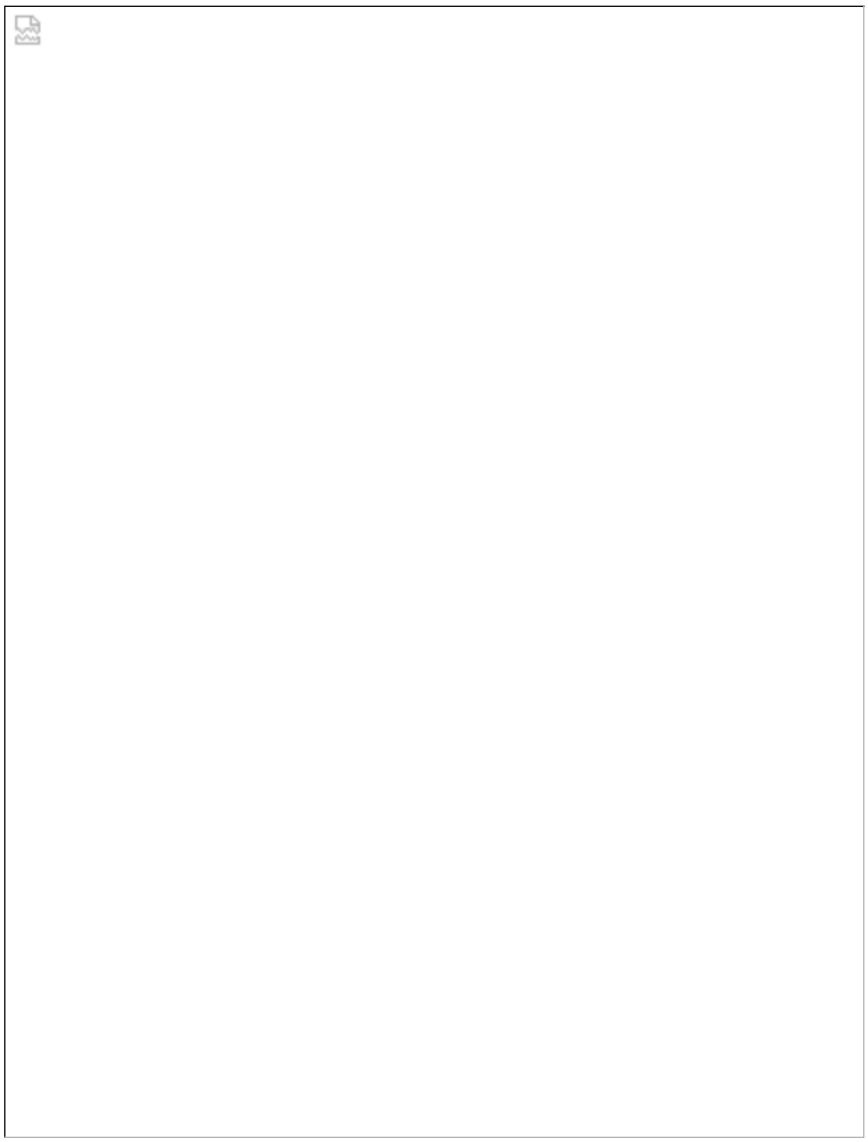
# Constantin Carathéodory



# Life and Work

by

***Roland Bulirsch* and *Michael Hardt***

**Guest lecture held at Vissa Orestiada, Greece in honor of Constantin Carathéodory**

**September 1-4, 2000**

Constantin Carathéodory was an extraordinary mathematician from the first half of the 20th century and probably the one Greek mathematician who has had the greatest influence worldwide.

We begin at a point in the middle of his life. In 1916, Carathéodory was a full professor of mathematics in Göttingen, Germany. At that time, he received this letter from Berlin.

**Figure 1:** Letter from Einstein to Carathéodory (undated).

*Berlin, Sunday*

*Dear colleague!*

*I find your derivation wonderful, now I understand everything. At first, the small writing mistakes on the second page had caused me some difficulties. Now, however, I understand everything. You should publish the theory in this new form in the Annals of Physics since the physicists do not normally know anything about this subject as was also the case with me. With my letter I must have come across to you like a Berliner who had just discovered Grunewald and wondered whether people were already living there.*

*If you wouldn't mind also making the effort to present to me the canonical transfromations, you'll find in me a grateful and attentive audience. If you, however, answer the question about the closed time trajectories, I will appear before you with my hands folded. The underlying truth, though, is well worth some perspiration.*

*Best regards, your Albert Einstein.*

Albert Einstein with his hands folded in front of Carathéodory. Those weren't just written words. Einstein's letter expresses the general sentiment towards Carathéodory at the time. At the acceptance of Carathéodory into the Prussian Academy of Sciences in Berlin in 1919, no one other than Max Planck gave the honorary dedication. In the previous year, 1918, Carathéodory had returned again to his birthplace, Berlin. What paths had taken him from Berlin to Berlin, and what an unusual life he had led.

On the 13$^{th}$ of september, 1873, he was born in Berlin as the son of a Turkish diplomatic attaché. Just one year later, his parents returned to the Porte in Constantinople. Shortly thereafter in 1875, his father became the Turkish ambassador in Brussels. Carathéodory's ancestors previously had had high state positions. A great uncle of his, Alexander Carathéodory Pasha, was a Turkish ambassador in Rome. Later he became the foreign minister, and he represented Turkey as the Turkish delegate to the Berlin Congress in 1878. There, the Turkish foreign minister Carathéodory Pasha did a lot for Greece; the Congress then `invited' the Ottoman Empire to revise its frontiers in favor of Greece which was not well received by the Porte in Constantinople. After his return from Berlin, he was dismissed from his post as the Turkish foreign minister. Yielding to external pressure, though, the Turks finally agreed in 1881 to cede Thessaly and the Arta district of Epirus to Greece.

Constantin Carathéodory grew up in Brussels under the care of his grandmother with Greek and French as his languages. He didn't learn German until later from a German instructor. His great grandparents lived in Marseille, and the young Carathéodory met there many of his relatives, from both his father's and mother's sides, who were spread over the entire European continent. He went to school in Brussels. In 1886, when he was thirteen years old, he was sent to a French high school, *Athénée Royal d'Ixelles*. In his geometry

class he discovered his love for mathematics, and he won at the French school system's usual competitions, the *Concour generaux*, several times the first prize in mathematics.

**Figure 2:** Carathéodory's military cadet institute.

In 1891 he completed his final high school examination, and he entered as an *élève étranger* into the *École Militaire de Belgique* which was a type of military cadet institute. He stayed there four years. The students were drilled like soldiers. The day began at five in the morning, and the instruction included military exercises, horseback riding, and physical exercise. The technical instruction was given by officers specialized in fortification engineering who had much experience in construction. Carathéodory could only express praise for the school. With projective geometry, he learned how valuable a geometrical perspective can be which, when treating it as a type of game, could help solve a wide range of problems. He valued highly his lectures on mechanics and thermodynamics. Carathéodory had many friends in the school, and he always continued to visit his friends in Belgium. Forty years later in 1936, he was already a "Geheimrat" (privy councillor), a highly respected title in Germany at the time. Even the Nazis would not consider interfering with those bearing the title. He met then for the last time his old friends in Belgium. Several had already been promoted to commanders in the army corps, general inspectors of artillery and fortification engineering. His good old friend Neefs had become a general and was then the director at his old military school.

In 1895, the young Carathéodory made his way with his officer's patent (civil engineer with officer's status) to Mytilene (Lesbos). His cousin Aristarchi was the engineer of the province and had built there the entire road network. Carathéodory helped him in planning the streets of Samos. The project's completion, though, was prevented by the Greek-Turkish war of 1896/97. Carathéodory moved on to London, and shortly thereafter in 1898, he went to Egypt, to Assuan und Assiout. In Assiout he worked for two years as an assistant engineer with the construction of dams for the Nile government. Day and night there was digging and construction taking place. Carathéodory spent many nights on the bottom of the Nile which had been pumped dry. In the evening and night by the intense heat he read mathematical text books. The lectures on analysis by Camille Jordan was one he particular loved. On the side, he also composed a work on the Cheops pyramid.

In the year 1900, Carathéodory decided to return again to Europe in order to dedicate himself completely to mathematics. His family and his Greek friends found his intention of leaving a secure, well-respected position which held much promise for Carathéodory in order to satisfy a romantic urge, as Carathéodory called it, not only just strange. They were shocked and horrified, and even Carathéodory himself was not convinced that he would be successful. He was driven, though, with the conviction that only his occupation with mathematics would give his life direction and meaning. All what remained open for him was where he would study. Should he go to Paris? That would have been the most natural thing to do since he had grown up in the French culture. Or should he maybe go to Berlin? In his acceptance speech of 1919 in the Prussian Academy of Sciences he spoke about this decision. *"In our house for more than 60 years, there has been a hand-carved picture of Alexander von Humboldt which I still proudly keep in the room in which I work. A tradition has thus stayed alive in me which, practically unknown to me, has led me to the place where this venerable, old prince of European intellect had completed the sum of his life´s accomplishments."*

In 1900, Carathéodory was in Berlin. He followed the lectures of the mathematician Frobenius with great enthusiasm, but he preferred to collaborate with Herrmann Amandus Schwarz, the successor of Weierstraß. He learned with him and from him function theory. Carathéodory himself discovered, and he repeated it many times, that one can understand in mathematics a general theory in the best way by mastering in depth some special example problems.If one could only place his faith in that, then almost the entire struggle of today's mathematical instruction could be explained.

Carathéodory became friends with Erhard Schmidt and Fejér. In 1902, Carathéodory moved to Göttingen, which was the stronghold of mathematics at the time, shining under the light from the mathematical double sun of Felix Klein and David Hilbert. He frequently visited in father in Brussels, and occasionally he traveled to his brother Telemachos who was director of the canal at Corinth. There on the Saronian sea, Carathéodory wrote his first mathematical work, *The theory of characteristics of partial differential equations of first order*. A Göttingen lecture from Hans Hahn from Vienna about the second variation was what stimulated him to study the problem of variational calculus. Carathéodory stated: a lamp which is surrounded by a semi-circular shaped globe projects points from the globe onto the ground. A curve of a given length is sought on the globe such that its shadow on the ground is as long or as short as possible. He found the solution: Two lines which meet in a corner, and only a short time later he completed his doctoral thesis on *Discontinuous Solutions in the Calculus of Variations*. He handed over his dissertation to Herrmann Minkowski, a founder of special relativity theory, and he passed soon thereafter the Rigorosum. He was tested in applied mathematics by Felix Klein and in astronomy by the not lesser known Schwarzschild.

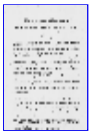


**Figure 3:** First page of Carathéodory's doctoral dissertation.

In 1903, Carathéodory was not interested in staying any longer in Germany and wanted to leave the country. Perhaps he felt that he was indeed more Greek and French. Then Felix Klein suggested to him that he do his Habilitation in Göttingen. That conversation with Klein laid out his destiny for the remainder of his life. David Hilbert urged him to immediately begin writing his Habilitation work, and the department of philosophy permitted him, on the request of Hilbert, to submit his Habilitation work immediately after the receival of his doctor´s degree.

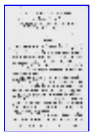


**Figure 4:** First page of Carathéodory's Habilitation.

**Figure 5:** Carathéodory in the year 1905.

He stayed for five years as a lecturer in Göttingen. He met Ludwig Prandtl, Herglotz, Toeplitz, and also Koebe again. The mathematician Runge especially impressed him. Carathéodory once commented, *The way in which Runge can work with mechanics is amazing. When the Wright brothers made their first attempts at flight, with the help of models that he made out of bits of paper attached by needles and dropping them so that they would glide, Runge was able to very closely estimate the motor's power whose data had been kept secret. This ability was what most impressed me. Additionally, he was also a first-class pure mathematician."*

However, in the meantime, there appeared another star in the sky above mathematics: *Carathéodory*.

In 1908, he switched to being a lecturer in Bonn. A year later in 1909, he became a full professor at the Technische Hochschule in Hannover, and in the following year, he was requested to come to the newly founded Technische Hochschule in Breslau, Silesia. Göttingen finally retrieved him in 1913, as the successor to the great Felix Klein.

In 1918, he received an offer to go to Berlin. When Carathéodory finally left Göttingen, in his farewell ceremony the Göttingen students and teaching assistants dedicated to him a long poem. The first lines when translated into English read something like this: *"Carathéodory is moving to Berlin and if this wouldn't be Göttingen, we would follow him."*

**Figure 6:** Carathéodory around 1920.

Two years later in 1920, he left Berlin again. Carathéodory followed a call from the Greek government. They wanted to start a university in Smyrna which Carathéodory could completely structure himself. The idea of an especially unique university captured him, a university which would "unify" the morning land (orient) and the evening land (occident). Oriental and occidental thought would have a common retreat, whatever that might mean today. Carathéodory stayed two years in Smyrna, and it all ended in a catastrophe. All Greek people know about it.

Carathéodory must have expected something terrible, and he probably knew more than the others around him. For the multilingual Carathéodory (Carathéodory spoke among all Western languages also Turkish), there were surely many things clear to him. Even before the Turkish attack, he had brought his family, his wife and two children, to safety on the neighboring Greek island of Samos. He stayed in Smyrna, but he had everything prepared for an evacuation, and he withstood the confusion of war until the last moments. While the Turks were already in Smyrna, he cooly brought his belongings and valuable writings to safety in Greece on boats. His own life was in great danger, but he was able to save himself from the devastated, burning Smyrna. The former soldier and officer Carathéodory saved the mathematician Carathéodory.

He found his retreat at the University of Athens. He held seminars within Greek mathematical circles. He spoke there about mathematical instruction in the higher classes, conducted reviews on Greek mathematical books, and he worked on an axiomatic structure for Einstein's relativity theory. It wasn't a very good time, though, for Carathéodory. Finally, Germany and Bavaria were able to bring him back. In 1924, he became the successor to Ferdinand Lindemann at the University of Munich. In 1925, he was elected to being an official member of the Bavarian Academy of Sciences. A stroke of luck for Bavaria and Germany.

**Figure 7:** Carathéodory in Pullach, Bavaria.

Once again he received a call from the Greek government. His Munich colleague, the great physicist Sommerfeld who was a close friend of Carathéodory, pleaded him not to go. "He should not have to once again place a crown of thorns upon his head," said Sommerfeld. In spite of it, in 1930, Carathéodory left duty-bound again for Greece, but he stayed only a few months in Athens. He occupied himself with the reorganization of the universities of Athens and Saloniki, and he completed his memoirs. He also wrote an article on mathematics for the large Greek encyclopedia. At the Acropolis, he investigated the Parthenon. He measured the curves at the base and the separation of the columns. The result of his measurements: the curves of the east and west sides of the temple are given with high precision from circles with radii of 1850 m. The curves of the north and south sides are given by circles whose radii are exactly three times greater, 5550 m.

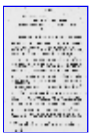


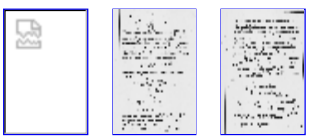
**Figure 8:** Mathematical investigation of the Parthenon.

Carathéodory had accomplished phenomenal feats in mathematics.

He worked on the calculus of variations, real-valued functions, function theory, measure theory, and the algebraization of the integral. He accomplished breakthroughs which were sensations in the mathematical field. In his works, highly creative space perspectives with a deep power of abstraction come together, and they are all masterfully written. He also worked on thermodynamics, geometrical optics, and the Schmidt reflective telescope. He personally conducted extensive numerical computations, and he calculated the diffractive curves from the "eikonal". The works were highly recognized by colleagues in physics.

His special love was for variational calculus. The great Max Planck said in 1919 to the Prussian Academy of Sciences: *"You, Mr. Carathéodory, have pointed out the double attraction which lies inside variational calculus. It shifts the view from difficult, confusing individual points to a clear, wide view of the whole. It can compose a wealth of individual statements together into one simple sentence, and even stranger, not only does man but also nature profit from this special manner of viewing things. I hope there will yet arrive many fruits from your scientific activities to decorate our academic journals."*

Here also belongs the previously mentioned letter of Einstein which was part of their correspondence about Hamilton-Jacobi theory. Albert Einstein had then around 1912 set aside his thoughts and ideas regarding general relativity theory and hoped with the help of the tools of Hamilton-Jacobi to arrive at deeper insights. With this in mind, on the 6$^{th}$ of September of 1916 he wrote to Carathéodory. At the end of his letter, Albert Einstein asked Carathéodory: *"Would you think a little bit about the problem of closed time trajectories? Here lies the essence of this still unsolved part of the space-time problem. I wish you all the best from yours truly, A. Einstein."*



**Figures 9-11:** Letter from Einstein to Carathéodory dated September 6, 1916.

Carathéodory answered on December 16, 1916: "*Dear colleague, the main points in the theory of canonical substitutions can be most easily derived in my opinion in the following way.*"There then comes mathematical expressions from Hamilton-Jacobi theory. The composition ends ... "*With best wishes, yours truly, C. Carathéodory.*"

**Figures 12-16:** Letter from Carathéodory to Einstein dated December 16, 1916.

Albert Einstein must then have sent his answer to the above-mentioned letter of Carathéodory, but it isn't dated (**Figure 1**). There then existed an exchange of letters between Carathéodory and Einstein. In this way Carathéodory communicated to him in 1925 how Blumenthal, who was later murdered, had been so extremely overjoyed about the present given to him on his 50th birthday. In 1928, several letters were sent to Einstein describing the extremely difficult relationship between the mathematicians Brower and Hilbert, both of whom were famous and active on the staff of the *Annals of Mathematics*. One can read about the hostility between Brower and Hilbert and can learn about the insults Hilbert made to Brower. This was an extremely delicate issue, and the time has still not come to shed light on such topics. The letter exchange between Einstein and Carathéodory is kept safe in the Einstein center in Jerusalem.

**Figure 17:** Equivalence of Carathéodory's equation to Bellman's equation.

The new field concept which Carathéodory introduced into variational calculus had profound consequences. In 1930, Carathéodory derived from it an inequality which 20 years later, under the different name of the Bellman equation or inequality, would become a sensation in the mathematical world. It provided the foundation for the principle of *Dynamic Programming* and has since extended far beyond mathematics. Bellman first presented his work in public later, after the death of Carathéodory, in 1951. The search for Carathéodory's name in these works is fruitless. This is one of the greatest injustices of modern scientific history. If Carathéodory had back in 1900 decided for France and Paris, this would never have happened. For a "French" Carathéodory, all of France's mathematicians would have caused an uproar, and the even extremely influential Bellman would never have dared to "mistreat" such a French mathematician. Bellman's impressive, indisputable accomplishments consisted of recognizing the great practical significance of Carathéodory's inequality and then using it for concrete computations. Carathéodory also wrote the introduction to Euler's works on variational calculus. Andreas Speiser from the Eidgenössischen Technischen Hochschule Zürich wrote about Carathéodory: "*The spirit of this great man and scholar continues to live in this historical and factual introduction, and it makes a very special adornment to our editon of Euler.*"When Alfred Pringsheim, the father-in-law of Thomas Mann, was forced in 1939 to leave Germany (was still allowed to leave), he gave as a present to Carathéodory a small token, a very rare text from Jacob Bernoulli to his brother Johann with the solution to the isoperimetric problem. Pringsheim dedicated this composition to his dear friend Carathéodory with the French play of words, "*Isopérimaître incomparable.*"How true: *Carathéodory, the imcomparable master*.

**Figure 18:** Carathéodory's introduction to Euler's works on variational calculus.

The Bavarian Academy of Sciences have published Carathéodory's collected works. It spans 5 volumes. The underlying inspiration came from his Munich colleagues: the "Geheimräte" (privy councillors) Tietze, Perron, and Sommerfeld. Mathematics professors from both of Munich's universities have supervised the revisions. Doctor Stephanos Carathéodory, the son, had translated for the volumes several of his father's works written in Greek into German.

## Final Comments

Carathéodory inherited a talent for languages which extends over generations in his family. Greek and French were his first languages, and he mastered German with such perfection, that his writings composed in the German language are stylistic masterworks. Carathéodory also spoke and wrote English, Italian, Turkish, and the ancient languages without any effort. Much more than that, Carathéodory was a treasured conversation partner for his fellow professors in the Munich Department of Philosophy. The well-respected professor for ancient languages Kurt von Fritz praised Carathéodory, saying that from him one could learn an endless amount about the old and new Greece, the old Greek language, and Hellenic mathematics. The philosopher Kurt von Fritz had an uncountable number of discussions with Carathéodory. Deep in his heart, Carathéodory felt himself above anything "Greek." The Greek language was exclusively spoken in the Carathéodory's house. His son Stephanos and daughter Despina went to a German high school, but they obtained daily additional instruction in Greek language and culture from a Greek priest. At home, they were not allowed to speak any other language.

The Munich-Augsburg evening newspaper printed Felix Klein's obituary written by Carathéodory. In Felix Klein's memory Carathéodory said, *"The strength of mathematics multiplies, like the giant Antaeus, when it makes contact with reality, the ground upon which it was grown."* The birthday wishes for the 80$^{th}$ birthday of his friend Alfred Pringsheim can be found in Munich's news. Carathéodory's composition out of the Deutsche Allgemeine Sonntagszeitung newspaper of April 1929 regarding "German Knowledge and its Worth" can still today wake up sad sentiments of nostalgia.

Carathéodory's language is that of a gentleman. This excerpt is out of his writings to the unfortunate Georg Cantor on his 70$^{th}$ birthday in 1915:

*Highly respected Sir!*

*We, who have so often made use in our work of your accomplished tools, would like to give to you our long overdue gratitude on today's festive day. Whoever has endeavored to enter into your teachings has witnessed something illustrious. Please accept our homage.*

Erhard Schmidt, himself an important mathematician, once said Carathéodory was completely free of the widespread faults of vanity and jealousy found frequently in the "academic world." *"He felt pure joy for others who made great accomplishments."*

**Figure 19:** Carathéodory around 1940.

Carathéodory was reclusive during the dark time in Germany. He was the head of a church counsel for the Greek church of the Savior on Munich's Salvatorplatz. This church had been given to the Greek orthodox community by the Bavarian king Ludwig I, who greatly admired Greece. In the meantime, Carathéodory was well over 60 years old and plagued by sickness. With his many connections in all parts of the world, though, he was able to give several of his "non-Arian" colleagues a chance for a future by arranging for them an opportunity to emigrate. His colleagues Tietze, Perron, and Sommerfeld probably kept from him many bad experiences they had had. Their gentlemanly conduct in Germany's worst times represents a noble distinction for the Bavarian Academy of Sciences.

Albert Einstein, in a letter from 1946 to Carathéodory's close friend, the phycisist Sommerfeld, *"Dear Sommerfeld, it was a true joy for me to receive your lively written words after all the dark years that we had to experience. We could never have imagined how things would turn out. It was a pleasure to hear that you belong to the few who remained intact."*

In December of 1949, Carathéodory gave his last talk in the Munich mathematical colloquium about lengths and surfaces. Shortly thereafter he became severely ill. He died on February 2, 1950. He was buried in the Munich Forest Cemetery in field number 303.

In honor of Carathéodory, there appeared a few years ago in the USA a two-volume collection with articles of prominent mathematicians from all parts of the world. The writings demonstrated the great influence of Carathéodory's thoughts and ideas in today's mathematics.

Carathéodory, Greece's present to Germany.

**Figure 20:** Greek stamp in honor of Carathéodory.

Greece had finally remembered their great son, the greatest Greek mathematician since ancient times. The Greek post office, in his honor, brought out a special stamp in 1994 portraying Carathéodory with formulas from variational calculus. Another stamp followed showing Thales von Milet. The auditorium of the new university in Xanthe, Thrachia, is named after Carathéodory, and it is otherwise well-known to the Greeks today what an extraordinary figure Carathéodory was.

Nobody could have said it as well as another famous member of the Bavarian Academy of Sciences , the Geheimrat Oskar Perron: *"Carathéodory, one of the most magnificent mathematicians, substantially enriched and vitally influenced the sciences ... a man of unusually extensive education. As a member of the Greek nation, with his soaring spirit and restless pursuit, he continued the recognition of the tradition and legacy of classical Greek culture."*

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## Sources:


[1] Constantin Carathéodory: *Gesammelte Mathematische Schriften I-V*. München, Beck 1957.

[2] Erhard Schmidt: *Constantin Carathéodory*, Volume V, see above.

[3] Oskar Perron: *Constantin Carathéodory*. Jahresberichte der DMV 55, pg. 39-51, 1952.

[4] Letters: Einstein-Center, Jerusalem.

[5] Personal dispatch from Frau Despina Rodopoulou-Carathéodory


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## Figures:

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